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The flavor distribution of Cosmic Neutrinos†.

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Abstract

For cosmic neutrinos, assuming that the neutrino mixing angles lie in the vicinity of their experimentally favored values, we derive simple analytical expressions for their relative flavor fluxes on Earth, in terms of the fluxes at the cosmic sites. This enables to disentangle clearly the sensitivity to the initial production fluxes as well as to small variations of the mixing angles. Such expressions should be useful in facilitating the analysis of the physical properties of cosmic neutrino production sites.

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The relative flavors of energetic Neutrinos reaching the Earth, after they have been emitted at various cosmic sites, provide useful information on the physical conditions there. Such sites may consist of exploding Supernovae creating neutrinos with energies at the ten-MeV range [1], or more energetic extragalactic sites like Gamma Ray Bursts and Active Galactic Nuclei (AGN), that may produce neutrinos that can reach the 10^3TeV [2], or even the 10^6TeV scale [3, 4, 5, 6]. Galactic candidates that may emit neutrinos of up to 100 TeV have also been identified at distances of at least 2.6kpc [7, 6].

It is commonly believed that, these neutrinos are produced mainly through the decay of high energy $\pi^\pm$ and K mesons, which implies that the initial relative neutrino flavors at the cosmic site satisfy $F^0_\mu/F^0_e \simeq 2$ and $F^0_\tau \simeq 0$, [4, 5, 6]. It may be useful to remember though that our present understanding of the mechanisms for generating high energy neutrinos is rather primitive, and sites may exist in the Universe where the produced neutrinos have a different initial structure [3, 8]. The measurement, therefore, of the relative intensities of the various neutrino flavors on Earth, should provide useful direct information on the mechanism responsible for their generation in the Cosmos.

Once the various neutrino flavors appear at the surface of some cosmic object, they propagate oscillating through space, following the vacuum oscillation formalism. It will therefore be useful to have simple formulae giving the observable relative numbers of neutrino flavors $F_e,~F_\mu,~F_\tau$, in terms of the initially produced ones $F^0_e,~F^0_\mu,~F^0_\tau$ at the surface of the various cosmic objects. Here we present such formulae, assuming only three active neutrino flavors[1] which propagate oscillating among themselves. In particular no cosmic neutrino decay is assumed [9].

For deriving the aforementioned formula, we take into account the basic experimental characteristics of the neutrino masses and mixings. These are summarized as follows: The recent SNO [10] data combined with those of Super-Kamiokande [11] strongly favor the LMA MSW [12] solar solution with three active neutrinos and $\theta_{12} \simeq \pi/6$ and $|m_2^2 - m_1^2| \simeq 5 \times 10^{-5}\, eV^2$ [13]. The atmospheric neutrino [14] data imply $\theta_{23} \simeq \pi/4$ and $|m_3^2 - m_2^2| \simeq 2.5 \times 10^{-3}\, eV^2$; while the CHOOZ experiment constrains $\theta_{13} \lesssim 0.2$, [15]. Defining then in the standard notation [16]

$$s_{12} \equiv \sin\theta_{12} \equiv \frac{1}{2} + \delta s_{12} \quad , \quad s_{23} \equiv \sin\theta_{23} \equiv \frac{1}{\sqrt{2}} + \delta s_{23} \quad , \tag{1}$$

we find [13, 14, 16],

$$-0.04 \lesssim \delta s_{12} \lesssim 0.19 \quad , \tag{2}$$

$$-0.15 \lesssim \delta s_{23} \lesssim 0.15 \quad , \quad -0.2 \lesssim s_{13}\cos\delta \lesssim 0.2 \quad , \tag{3}$$

where $s_{13} \equiv \sin\theta_{13}$.

For realistic neutrino mass differences, and neutrino energies in the range $E \lesssim 10^6$ TeV, the vacuum oscillation lengths $\lambda_{ij} = 4\pi E/|m_i^2 - m_j^2|$, always satisfy $\lambda_{ij} \lesssim 1\,\mathrm{pc}$, which is much smaller than the distances to all cosmic neutrino emitting sites, beyond our solar system [7]. Consequently, the number of oscillations performed by the cosmic neutrinos

[1]See *e.g.* [16].

before arriving at the Earth, is so large, that $\sin^2(\pi L/\lambda_{ij})$ average to 1/2, and the CP-violating contributions vanish.

Expanding the standard vacuum oscillation formulae to first order in δs_{12}, δs_{23} and s_{13}, we get for the induced neutrino relative flavors on Earth,

$$\begin{aligned} F_e &= \frac{3+7F_e^0}{16} - \frac{\delta s_{12}}{2}(3F_e^0 - 1) + \delta s_{123}(F_\tau^0 - F_\mu^0) \ , \\ F_\mu &= \frac{13-7F_e^0}{32} + \frac{\delta s_{12}}{4}(3F_e^0 - 1) + \delta s_{123}(F_\mu^0 - F_e^0) \ , \\ F_\tau &= \frac{13-7F_e^0}{32} + \frac{\delta s_{12}}{4}(3F_e^0 - 1) + \delta s_{123}(F_e^0 - F_\tau^0) \ , \end{aligned} \tag{4}$$

where

$$\delta s_{123} \equiv \frac{\sqrt{3}}{8}(\sqrt{6}\delta s_{23} - s_{13}\cos\delta) \ , \tag{5}$$

and F_e^0, F_μ^0, F_τ^0 are the initial neutrino relative flavors at the cosmic site. In writing (4) we took into account the unitarity relation

$$F_e + F_\mu + F_\tau = F_e^0 + F_\mu^0 + F_\tau^0 = 1 \ , \tag{6}$$

where the right hand side is just a normalization. Equation (4) is our basic result. It depends on two mixing angle parameters only; namely δs_{12} which is experimentally constrained by (2), and the combination δs_{123} for which (3) implies

$$-0.12 \lesssim \delta s_{123} \lesssim +0.12 \ . \tag{7}$$

We next turn to the discussion of three interesting specific cases.

Because of unitarity, if the initial relative flavors satisfy $F_e^0 = F_\mu^0 = F_\tau^0$, then the final ones also obey $F_e = F_\mu = F_\tau = 1/3$, irrespective of the neutrino mixing angles. This is the situation expected *e.g.* for neutrinos (or antineutrinos) generated in supernovae explosions [1]. We call it "Supernova-type case", allowing it to cover also the possibility of TeV neutrino sources which somehow produce equal neutrino fluxes for all neutrino and antineutrino flavors.

High energy neutrinos are commonly assumed to be generated in connection to the high energy cosmic rays, through some beam-dumb process producing unstable mesons (mainly $\pi^\pm$), which subsequently decay while crossing regions of space with rather small energy density. In such regions it is then expected that $F_e^0 = 1/3$, $F_\mu^0 = 2/3$, $F_\tau^0 = 0$ [4, 5, 6, 3, 17], which leads to

$$\begin{aligned} F_e &= \frac{1}{3}(1 \ - \ 2\,\delta s_{123}) \ , \\ F_\mu = F_\tau &= \frac{1}{3}(1 \ + \ \delta s_{123}) \ . \end{aligned} \tag{8}$$

This situation we call "canonical case". The result (8) agrees with the conclusion of [5, 18] that for bimaximal neutrino mixing with very small s_{13}, the relative neutrino flavor fluxes are $F_e \simeq F_\mu \simeq F_\tau \simeq 1/3$. Our formalism goes beyond this though, since it predicts that the arriving neutrino fluxes are independent of δs_{12}, and that they only depend on the specific combination of δs_{23} and $s_{13}\cos\delta$ entering (5).

We thus find from (8) and the constraint (7), that all relative neutrino fluxes on Earth should satisfy

$$\begin{aligned} 0.25 \lesssim F_e &\lesssim 0.41 \quad , \\ 0.29 \lesssim F_\mu = F_\tau &\lesssim 0.37 \quad , \end{aligned} \tag{9}$$

while for ν_μ/ν_e number ratio we get

$$0.6 \lesssim \frac{F_\mu}{F_e} \simeq (1 + 3\ \delta s_{123}) \lesssim 1.4 \quad . \tag{10}$$

It is worthwhile to note that the ranges in (9, 10), which have been derived analytically, are very similar to those derived in the numerical analysis of [5, 18]. A virtue of the present derivation, is that the effect of a future reduction of the experimental uncertainties on the mixing angles, can be straightforwardly read from (8).

As an example, we note that if it turns out that *e.g.* $\delta s_{123} = 0.1$ (compare (7)), then (8) would imply $F_e = 0.27$, and $F_\mu = F_\tau = 0.37$; which, in a future sufficiently large neutrino telescope, might be possible to discriminate from the "Supernova-type" case predicting $F_e = F_\mu = F_\tau = 0.33$.

As a next rather exotic situation we consider the case $F_e^0 = 1$, $F_\mu^0 = F_\tau^0 = 0$. In this case

$$\begin{aligned} F_e &= \frac{5}{8} - \delta s_{12} \quad , \\ F_\mu &= \frac{3}{16} + \frac{\delta s_{12}}{2} - \delta s_{123} \quad , \\ F_\tau &= \frac{3}{16} + \frac{\delta s_{12}}{2} + \delta s_{123} \quad , \end{aligned} \tag{11}$$

where, in contrast to the previous situation, the relative neutrino fluxes have some sensitivity to δs_{12} also. Using (2, 7) we then find

$$\begin{aligned} 0.44 \lesssim F_e &\lesssim 0.67 \quad , \\ -0.24 \lesssim F_\mu - F_\tau &\lesssim 0.24 \quad , \end{aligned} \tag{12}$$

in which the presentation has been chosen so that the uncertainties induced by δs_{12} and δs_{123}, are separated.

In the present paper we have assumed just three neutrino flavors that propagate in space oscillating among themselves, without the presence of any sterile neutrinos or neutrino decay processes[2]. Assuming then that the deviations of the neutrino mixing angles

[2]The above formulae can of course straightforwardly be extended to cases including sterile neutrinos.

from their "canonical values" $s_{12} = 1/2$, $s_{23} = 1/\sqrt{2}$ and $s_{13} = 0$ are small, we expressed the observable neutrino fluxes on Earth, in terms of the original ones at the cosmic sites, keeping only linear terms in the aforementioned angle-deviations.

The simplicity of these expressions should render them useful in the analysis of future Neutrino Astronomy data. In particular they may help us in performing cosmic scans using the physical properties of the neutrino fluxes arriving from various directions of the Universe. They may thus facilitate the analysis of the physical properties of some intriguing cosmic objects.

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